The family as producer of health — an extended grossman model

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Received 7 May 1998; received in revised form 24 February 1999; accepted 30 November 1999

Abstract

Deriving a model, where each family member is the producer of his own and other family members’ health, shows that the family will not try to equalise marginal benefits and marginal costs of health capital for each family member. They will rather invest in health until the rate of marginal consumption benefits equals the rate of marginal net effective costs of health capital. The level of compensation in the social insurance system, the effective price of care, health related information, and transfer payments will all affect the production possibility set, and therefore the optimal level and distribution of family health.

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JEL classification: I1; I12

Keywords: Health; Human capital; Family; Grossman model

1. Introduction

In his seminal work, Michael Grossman (1972a,b) constructed and estimated a dynamic model of the demand for the commodity ‘good health’. He argued that
such a model is important for two reasons. First, because the level of health influences the amount and productivity of labour supplied to an economy. Second, what consumers demand when they purchase medical services are not these services per se but rather ‘good health’. Added to these, there are further reasons. The model will help explain individuals’ health related behaviour, e.g., why some people smoke and some not, and why different individuals have different health and different health care utilisation (Muirinen and Le Grand, 1985). In order to evaluate and predict the effects of regulations, new technology, changes in social insurance schemes and government, and other programmes, knowledge about the effects on individuals’ demand for health and health related behaviour is essential. These effects will not be confined just to the present distribution of health capital (direct effects), but they will change the individual’s lifetime health profile (indirect effects). Hence, present and future total utilisation of health care resources and social insurance are to a large extent the result of aggregate previous, present and future health related behaviour. Therefore, increasing our knowledge of what factors determine observed inequalities in health and the path of lifetime health has important policy implications.

Fundamental to the demand for health model is the sharp distinction between market goods and commodities. In this approach, consumers produce commodities with inputs of market goods and their own time (Becker, 1965). For example, they use sporting equipment and their own time to produce recreation, travelling time and transportation services to produce visits, and part of their Sundays and church services to produce ‘peace of mind’. Since goods and services are inputs into the production of commodities, the demand for these goods and services is a derived demand (Grossman, 1972a, p. xv).

Further, the commodity good health is treated as a durable item, as one component of human capital. Health is demanded by the consumer for two reasons: as a consumption commodity, directly entering the individual’s utility function (i.e., sick days being a source of disutility); and as an investment commodity, determining the total amount of time available for market and nonmarket activities. According to the model, the level of health is endogenous, and depends (at least in part) on the resources allocated to its production. The shadow price of health will depend on many variables besides the price of medical care, and the quantity of health demanded will be negatively correlated with its shadow price.

Though the model yields valuable contributions to explain individuals’ health related behaviours and differences in health and health care utilisation, attempts to develop the theoretical model have been relatively few (for a review, see Grossman, 1982, 1998), and they have all been based on the individual as producer of health. This implies that both the original model, as well as the extended models, can only be used to analyse adult health, not children’s ‘demand’ for health and
their health care utilisation. It also implies that the influence of other family members on the individual’s demand for health and demand for health care cannot be considered.

However, to analyse health issues from a family perspective is important for several reasons. It has been suggested that events during fetal life and early childhood are associated with disease and mortality in later life (Barker, 1992, 1994; Power et al., 1996; Wadsworth, 1996, 1997). The pathway linking early life with adult disease is explained by the process of ‘biological programming’ (Barker, 1992, 1994) and the continuities in lifetime socio-economic circumstances. This implies that a life course perspective is needed when searching for the determinants of inequalities in health, and a demand for health model where the production of child health is included.

Further reasons for a family perspective, i.e., to include the influence other family members have on an individual’s health related behaviour, is supported by some empirical findings. Grossman (1975) found that an increase in wives’ schooling increased male health. Actually Grossman found the coefficient on wives’ schooling to exceed the coefficient of men’s own schooling (however, not significantly higher). Currie and Gruber (1996) examined the effects of public health insurance on children’s health and utilisation of medical care. They found that parents with some college education are more likely to take their children to the doctor, with a stronger effect for mothers than for fathers, and that utilisation is higher both for first children and for children in smaller families. Further findings are that, conditional on household income, children in households with no male head have higher utilisation levels, and that visits seem to be normal goods while hospitalisations appear to be inferior goods. Thomas et al. (1991) found that a mother’s education has a large impact on child height in Northeast Brazil, and that almost all of this impact can be explained by indicators of her access to information. Delaney (1995) reports on negative effects of parental divorce on children’s health. Parental divorce also seems to have long-term effects on personality and longevity (Tucker et al., 1997).

Furthermore, how resources are allocated into investments in formal schooling and child health, as well as into investments in adult health and on-the-job

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1 Previously, for example Grossman (1975; see also Leibowitz, 1974) has argued that the health and intelligence of children partly depend on genetic inheritance, but that in particular they depend on early childhood environmental factors, which are shaped to a large extent by parents. However, Grossman et al. (Becker, 1991; Cigno, 1991) view the increase in direct utility as the reason for parents’ investing in child quality, i.e., that child quality is a consumption commodity. This paper argues that there exists a monetary incentive as well.

2 See Becker’s (1991, Chapter 6) model of family background and the opportunities of children, where he analyses the influence family expenditures and endowments have on the income of children.
training, are decisions made jointly within the family. Existing demand for health models cannot explain and analyse such family and lifecycle related issues. Analysing health matters from a family perspective will be a step in the direction to a model in which the stocks of health and knowledge are simultaneously determined.3

This paper extends the Grossman model in that the family is seen as the producer of health4. By the family as producer of health is meant that each family member is the producer, not only of his own health, but also of the health of other family members, and that not only his own income and wealth, but also the earnings of other family members, can be used in the production of health. The model will be derived assuming complete certainty. The implications of relaxing this assumption will be briefly discussed in the closing section of the paper.

According to Grossman, the individual receives both investment and consumption benefits from investing in his own health. This paper argues that this is valid also for investments in other family members’ health. Investment benefits occur because increased adult health (or child health) will decrease future time spent sick (or time spent taking care of a sick child). Family time available for market work will then increase, which may raise family income and increase consumption and investment possibilities for all family members. Consumption benefits may also occur, if family members derive utility not only from own health, but from the health of other members as well, i.e., the individual cares about the well-being of his or her child and spouse as he or she cares about his or her own well-being.

Even though the individual may have incentives for producing health of other family members (both with and without partly altruistic preferences), it is not self-evident how the objective function should be treated. While there is only one person who maximises his or her lifetime utility in the traditional Grossman individual demand for health model, there are at least two persons with common or non-common interests to consider, when formulating the optimisation problem in the family version of the model. In the model of the family as producer of health developed in this paper, the family (rather than the individuals making up the family) is the economic unit, and a common preference approach will be used (Becker, 1974, 1991). The implications of relaxing this assumption will be briefly discussed in the final section of the paper.

Viewing the family as producer of health will not only affect how the benefits from investments in health are treated in the model, but also the household

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3 Currently, we still lack comprehensive theoretical models in which the stocks of health and knowledge are determined simultaneously. I am somewhat disappointed that my 1982 plea for the development of these models has gone unanswered [Grossman (1982)].” (Grossman, 1998, p. 5).

4 More exactly, parents are the producers of their own and their child’s health, while children are assumed passive.
production function(s) and the amount of available resources. In Grossman’s model, productivity is determined by the individual’s education. But seen from a family perspective, productivity may be determined by other family members’ education as well. Further, it may not be the individual’s education per se that determines his or her productivity in producing health, but human capital specific to that activity. Resources available for health production are not only own income, but total family income. However, an important difference between investing in own health and investing in another person’s health is the difficulty to observe when ‘enough’ investments have been made; being uncertain whether enough has been done to restore the health of the other person. In contrast, for investments in own health, one knows when one prefers to use time and money to produce other commodities than health.

To analyse an individual’s lifetime health and lifetime health care utilisation profile, it is necessary to use a lifecycle approach. A lifecycle approach may also be essential in order to explain observed differences in health and medical care utilisation among individuals. The lifecycle model concerns individual investment decisions and deals with resource allocation over an individual’s lifetime rather than solely with decisions of the present time period (see Polachek and Siebert 1993 for a description of the lifecycle human capital model, and how the lifecycle approach can be used to explain earnings variations). While an individual’s amount of education and on-the-job training determine the individual’s lifetime earnings, health capital determines not only the individual’s productivity but also his or her amount of healthy time. To tackle such a dynamic optimisation problem, there are three major approaches: calculus of variations, dynamic programming, and optimal control theory (Chiang, 1992, pp. 17–22). In this paper, optimal control theory will be used.

The paper is organised as follows. In Section 2, a model of the family as producer of health is derived in three steps. First, as a frame of reference, a model of the single person family is derived. Then, a model for the husband–wife family is derived and, eventually, a child is added to the family and a parents–child family model is presented. In Section 3, a graphic illustration of the family as producer of health model follows, and the effects on family health of changes in some exogenous variables are analysed. The paper concludes by a discussion of the implications of relaxing the assumption of common preferences and instead assumes allocation within the family to be the outcome of a cooperative Nash bargaining model, as well as a brief discussion of relaxing the assumption of complete certainty. Section 4 also includes some remarks regarding family forma-
tion, family size, and inter-sibling allocation, as well as a discussion of some policy implications.

2. The family as producer of health

In the following, a model of the family as producer of health will be developed successively in three steps. As a frame of reference, a model of the single person family will be presented, then the husband and wife family, and finally the parents and child family.

In all three models, the family is assumed to choose the amount of market goods to consume in each time period in order to maximise family lifetime utility, given initial family wealth and each family member’s initial amount of health capital, and given the production functions and prices. The time path of family wealth and the paths of each family member’s health are then given by the optimal amounts of market goods chosen.

2.1. The single-person family

Similar to Grossman (1972a,b), the individual is assumed to derive utility from own health, $H_t$, and from the consumption of other commodities $Z_t$. The individual has a strictly concave utility function, where utility in period $t$ is

$$ u_t = u(H_t, Z_t). \tag{1} $$

The individual’s stock of health will depreciate during his or her lifetime, but the individual can invest in health (produce health capital) to offset this depreciation in health capital. The individual’s stock of health will develop over time according to

$$ \frac{\partial H_t}{\partial t} = I_t - \delta_t H_t, \tag{2} $$

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7 The three versions of the family as producer of health model could be seen as tools for analysing health and health care utilisation during different stages in an individual’s life: as a child in a two parents’ family; as a single adult person (either prior to partnership, after a divorce, or at the end of life); as partners without children; and as partners with one child. The objective was not to analyse transitions between stages — that would have required quite a different analytical framework — and there are obviously many more possible stages to account for in real life. The objective was rather to make a model that could provide important insights into family health behaviour, given the fact that the family exists.

8 Those ‘other commodities’ are produced and consumed in period $t$, i.e., they cannot be stored.
which is the equation of motion for the state variable health, and where \( \delta_t \) is the rate of depreciation. The individual produces gross investments in health, \( I_t \), and other commodities, \( Z_t \), according to the production functions:

\[
I_t = ( M_t, h_{1,t}; E_{1,t} )
\]

and

\[
Z_t = ( X_t, h_{1,t}; E_{1,t} ),
\]

where \( M_t \) and \( X_t \) are market goods\(^9\), and \( h_{1,t} \) and \( h_{1,t} \) are own time used in the production of health and other commodities, respectively, \( E_{1,t} \) and \( E_{1,t} \) are efficiency parameters\(^10\), and the production functions are assumed homogenous of degree one in both goods and time inputs\(^11\).

Individual (family) stock of wealth \( W_t \) will develop over time according to

\[
\frac{\partial W_t}{\partial t} = r W_t + \omega_t (H_t, E_{1,t}) h_{1,t} + B_t - p_t M_t - q_t X_t,
\]

the equation of motion for the state variable wealth, where \( r \) is the market interest rate, \( \omega_t \) is the wage rate, \( E_{1,t} \) is the level of education and on the job training, \( h_{1,t} \) is time in market work, \( B_t \) is transfers, and \( p_t \) and \( q_t \) are the prices of medical care \( (M_t) \) and other goods \( (X_t) \), respectively.

Health will affect market income in two ways: through its effect on the wage rate; and through its effect on healthy time available for market work. According to the formulation in Eq. (5), an individual’s productivity in market work is determined by his or her amount of health capital and level of education and on-the-job training, implying that \( \omega_t (H_t, E_{1,t}) \) can be thought of as the ‘labour market earnings rate of return on human capital’.

The available amount of healthy time in each time period is total time \( (\Omega) \) less time spent sick \( (h_{1,S}) \), where time spent sick is determined by the individual’s amount of health capital \( (h_{1,S} = h_{1,S}(H)) \).\(^{12}\) Time spent in market work \( (h_{1,M}) \), in producing health \( (h_{1,H}) \) and other commodities \( (h_{1,Z}) \), and time spent sick \( (h_{1,S}) \) have to sum up to total time available \( (\Omega) \), i.e.,

\[
\Omega = h_{1,m} + h_{1,H} + h_{1,Z} + h_{1,S}.
\]

\(^{9}\)Note that inputs in the production of health may not only be medical care services, but diet, exercise, housing, etc. See Grossman (1972a), for an analysis of joint production. Here, however, it will be assumed that medical care services are the only inputs in the production of health.

\(^{10}\)In Grossman (1972a, b) the individual’s productivity is determined by his or her level of schooling. As in Becker (1991), individual productivity in producing different commodities may differ (activity specific human capital). The individual’s productivity in producing health may depend on information and knowledge about health matters as well as the individual’s stock of health, while the individual’s productivity in market work depends on his or her stock of educational capital (formal education and on-the-job training).

\(^{11}\)An assumption made by Grossman as well as his successors.

\(^{12}\)\( \partial h_{1,S} / \partial H_1 < 0, \partial^2 h_{1,S} / \partial H_1^2 > 0. \)
The individual’s problem is then to choose the time paths of the control variables \( M_t \) and \( Z_t \) that maximise lifetime utility. The problem can then be written:

\[
\text{Max } U = \int_t^T e^{-\theta u} u(H_t, Z_t)
\]

such that

\[
\partial H_t / \partial t = I_{t,H} - \delta H_t
\]

\[
\partial W_t / \partial t = r W_t + \omega_0 (H_{t,\omega}) h_{t,\omega} + B_t - p_t M_t - q_t X_t
\]

\[
\Omega = h_{t,\omega} + h_{t,H} + h_{t,Z} + h_{t,S}
\]

\[
H(0) = H_0, \quad W(0) = W_0, \quad H_0 \text{ and } W_0 \text{ given}
\]

\[
H(T) = H_T \leq H_{\text{min}}, \quad W(T) = W_T \geq 0, \quad W_T^b \lambda_{T, W} = 0
\]

T free

and \( X_t, M_t \geq 0 \) for all \( t \in [0, T] \) \hspace{1cm} (7)

where \( U \) is the individual’s intertemporal utility function, i.e., the discounted value of the individual’s lifetime utility, discounted by the individual’s rate of time preference, \( \theta \). \( \partial H_t / \partial t \) and \( \partial W_t / \partial t \) are the equations of motion for the state variables \( H_t \) and \( W_t \), respectively, and \( \Omega \) the time restriction. \( H_{\text{min}} \) is the individual’s ‘death stock’ of health capital. The individual dies when health passes below some level \( H_{\text{min}} \), which determines \( T \) (time of death). It should be observed that the individual is free to borrow and lend capital at each period, but the bequest \( (W_T) \) cannot be negative.

The solution to this horizontal-terminal-line problem (Chiang, 1992) gives that the individual invests in health until the marginal benefit of new health equals the marginal cost of health (see Appendix A):

\[
(\text{e}^{-\theta} / \lambda_{t}, W) \partial u_t / \partial H_t + h_{t,\omega} \partial \omega_0 / \partial H_t - (\varphi / \lambda_{t}, W) \partial h_{t, S} / \partial H_t
\]

\[
= \pi_t \left[ \delta_t + r - (\partial \pi_t / \partial t) / \pi_t \right], \hspace{1cm} (8)
\]

where \( \lambda_{t}, W \) (and \( \lambda_{t}, H \)) are costate variables, \( \varphi_t \) is the lagrange multiplier for the time restriction, \( \partial u_t / \partial H_t \) is marginal utility of health capital\(^{13}\), \( \partial \omega_0 / \partial H_t \) is the marginal effect of health on wage\(^{14}\), \( h_{t, S} / \partial H_t \) is the marginal effect of health on the amount of sick time and \( \pi_t \) is the effective price of medical care goods and services \( (M_t) \).

The first order condition (A10) in the Appendix A gives that \( \lambda_{t}, H \) equals \( \lambda_{t}, W \pi_t \). Thus, in periods when the budget is binding (\( \lambda_{t}, W \) high) or the effective

\(^{13}\) \( \partial u_t / \partial H_t > 0, \partial^2 u_t / \partial H_t^2 < 0 \).

\(^{14}\) \( \partial \omega_0 / \partial H_t > 0 \).
price of care ($\pi_t$) is high, $\lambda_{t,H}$ will be high, implying that the individual’s stock of health is low. Further, the first order condition (A7) shows that:

$$\frac{\partial \lambda_{t,H}}{\partial t} = \lambda_{t,H} \delta_t - \lambda_{t,W} h_{1,w} (\frac{\partial \omega_t}{\partial H_t}) + \varphi_t (\frac{\partial h_{t,s}}{\partial H_t}) - (\frac{\partial u_t}{\partial H_t}) e^{-r_t},$$

(9)
i.e., the time path of $\lambda_{t,H}$ will depend on, for example, the rate of depreciation in health ($\delta_t$), the sensitivity in the individual’s wage rate to changed health ($\frac{\partial \omega_t}{\partial H_t}$), and the individual’s valuation of time [how binding the time restriction is ($\varphi_t$)]. An increased rate of depreciation will increase $\lambda_{t,H}$, decreasing the individual’s level of health. If the individual’s wage rate becomes more sensitive to differences in health, the individual will invest more in health ($\lambda_{t,H}$ decreases). Finally, the more restricting the time constraint is, the higher will the individual’s valuation of time be ($\varphi_t$), and the more will the individual invest in health (decreasing $\lambda_{t,H}$).

Note that while $\frac{\partial u_t}{\partial H_t}$ is the increase in utility in period $t$ if health capital in period $t$ is increased by one unit, $\lambda_{t,H}$ is the increase in lifetime utility if health in period $t$ is increased by one unit of health capital.

Looking at the time path of the costate variable $\lambda_{t,W}$, the solution shows that $\lambda_{t,W}$ decreases over time with a rate equal to the rate of interest, $r$. According to the present formulation, the individual is free to borrow and lend capital at each period of time ($\frac{\partial W_t}{\partial t}$ can take both positive and negative values), but $W_T$ is

![Fig. 1. An illustration of the paths of the control variable, $c$, and the state variable, $y$, in an optimal control problem. (a) Illustrates that the control path, $c(t)$, does not have to be continuous; it only has to be piecewise continuous. For example, an individual’s medical care consumption can make jumps over time, i.e., be positive in some time periods and zero in some. The state path, $y(t)$, on the other hand, has to be continuous throughout the time period $[0,T]$ as illustrated in (b). However, the state path is allowed to have a finite number of sharp points, i.e., it needs to be piecewise differentiable. Those sharp points will occur at the times when the control path makes a jump. For example, a jump in medical care consumption, say at time $t_1$, may correspond to a sharp decrease in health, $y(t)$. To place a state-space constraint on the maximisation problem can, for example, be to only allow for positive values on $y(t)$, i.e., the permissible area of movement for $y$ is the area above the horizontal axis (for example, if $y$ is monetary wealth, to allow for saving but not for borrowing). Source: Chiang (1992, p. 163, Fig. 7.1).](image)
restricted to be non-negative, i.e., the bequest cannot be negative. If formulating
the problem as a state-space constraint problem (Chiang, 1992, pp. 298–300),
\( \dot{W} / \dot{t} \) is forced to be non-negative in every period. \( \lambda_{t,w} \) will then not be
continuous, but make jumps in periods where this restriction is binding (see Fig. 1
for an illustration).

The values of \( \lambda_{t,w} \) and \( \varphi_t \) may be interpreted as measures of stress, economic
and time stress, respectively. If the wealth and/or time constraint is binding
for several periods, the values of \( \lambda_{t,w} \) and \( \varphi_t \) will be high and increasing.

2.2. The husband–wife family

Using a common preference model of family behaviour, the instantaneous
family (strictly concave) utility function can be written as

\[
U = U(H_{m}, H_{t}, Z),
\]

(10)

where time subscripts are omitted in order to simplify the notations. \( U \) is family
utility in period \( t \), \( H_m \) and \( H_t \) are husband (male) and wife (female) health,
respectively, and \( Z \) is a vector of commodities consumed. As in the single-person
model, the depreciation in male (\( \delta_m \)) and female health (\( \delta_f \)) may be offset by
gross investments in male (\( I_m \)) and female (\( I_t \)) health, respectively, according to
the production functions:

\[
I_m = I_m(M_m M_{Hm,m}, h_{Hm,m}, E_{Hm,m}, E_{Hm,t})
\]

(11)

and

\[
I_t = I_t(M_t M_{Ht,m}, h_{Ht,t}, E_{Ht,m}, E_{Ht,t})
\]

(12)

where the production functions are assumed homogenous of degree one in goods
and time inputs. \( M_m \) and \( M_t \) indicate market goods used in the production of male
and female health, respectively. Time used in the production of health is indicated
by \( h_{Hm,m} \), \( h_{Ht,m} \), \( h_{Hm,t} \), and \( h_{Ht,t} \). The first subscript denotes what is produced;
male (\( H_m \)) or female (\( H_t \)) health, the second subscript denotes who is the
producer; the husband (m) or the wife (f). \( E_{Hm,m} \) and \( E_{Ht,t} \) indicate male and female
productivity in health production. Similarly, net investments in health are

\[
\delta H_m / \dot{t} = I_m - \delta_m H_m
\]

(13)

and

\[
\delta H_t / \dot{t} = I_t - \delta_t H_t.
\]

(14)

The development of family wealth follows

\[
\dot{W} / \dot{t} = rW + \omega_m(H_m, E_{w,m})h_{w,m} + \omega_t(H_t, E_{w,t})h_{w,t} + B
- p(M_m + M_t) - qX,
\]

(15)
where \( \omega_m(H_m,F_{w,m}) \) and \( \omega_f(H_f,F_{w,f}) \) are the husband’s and wife’s wage rates (or labour market earnings rates of return on human capital), respectively. \( E_{w,m}(F_{w,m}) \) is the husband’s (wife’s) level of education and on-the-job training, and \( h_{w,m}(h_{w,f}) \) his (her) amount of time spent in market work.

The time restrictions are

\[
\Omega_i = h_{w,i} + h_{Z,i} + h_{H_m,i} + h_{H_f,i} + h_{S,i}  \quad i = m,f.
\]

(16)

Total time for each spouse (\( \Omega_i \)) is allocated between time spent in market work \( (h_{w,i}) \), in home production of health \( (h_{H_m,i} + h_{H_f,i}) \) and other commodities \( (h_{Z,i}) \) and time being sick \( (h_{S,i}) \), where health determines the amount of sick time \( (h_{S,i} = h_{S}(H_j)) \).

The problem facing the family is to choose the time paths of the control variables \( M_m, M_f, \) and \( Z \), in order to maximise lifetime utility. The problem can then be written (time subscripts still omitted for simplicity):

\[
\text{Max } U = \int_0^T e^{-\beta t}u(H_m,H_f,Z) \\
\text{such that } \frac{\partial H_m}{\partial t} = I_m - \delta_m H_m \\
\frac{\partial H_f}{\partial t} = I_f - \delta_f H_f \\
\frac{\partial W}{\partial t} = rW + \omega_m(H_m,F_{w,m})h_{w,m} + \omega_f(H_f,F_{w,f})h_{w,f} \\
\quad + B - p(M_m + M_f) - qX \\
\Omega_i = h_{w,i} + h_{Z,i} + h_{H_m,i} + h_{H_f,i} + h_{S,i}  \quad i = m,f \\
H_m(0), H_f(0), W(0) \text{ given} \\
H_m(T) \quad \text{and/or} \quad H_f(T) \leq H_{\text{min}} \\
W(T) \geq 0, \quad W^T Y_f = 0 \\
T \text{ free} \\
\text{and } X, M_m, M_f \geq 0 \quad \text{for all } t \in [0,T] \quad (17)
\]

\( T \), in this case, is the ‘lifetime’ of the husband–wife family; the family ‘dies’ when husband and/or wife no longer has a health status greater than \( H_{\text{min}} \). The solution to this maximisation problem gives the marginal condition (see Appendix A, condition A16):

\[
\frac{\partial u}{\partial H_m} = \frac{\pi_m(\delta_m r + \frac{\partial \pi_m}{\partial t} / \pi_m) - [(\partial \omega_m / \partial H_m)h_{w,m} - (\varphi_m / \lambda_m)(\partial h_{S,m} / \partial H_m)]}{\pi_t(\delta_t r + \frac{\partial \pi_t}{\partial t} / \pi_t) - [(\partial \omega_t / \partial H_t)h_{w,t} - (\varphi_t / \lambda_t)(\partial h_{S,t} / \partial H_t)]}.
\]

(18)

Thus, the optimal condition (8), i.e., that the individual invests in health until marginal benefits equal marginal costs, is not valid any more. In a two-person family with common preferences, husband and wife together invest in health until the rate of marginal consumption benefits (left hand side of Eq. (18)) equals the
rate of marginal net effective cost of health capital (right hand side). The net effective cost of health capital equals the user cost of capital less the marginal investment benefit of health capital (in brackets). A similar result (see Eq. (A10) in Appendix A) can be derived for lifetime utility of health, i.e., that

\[ \lambda_w = \lambda_{H_{m}}/\pi_m = \lambda_{H_{f}}/\pi_f. \]  

(19)

Condition (19) implies that family members will invest in health until the rate of marginal lifetime utility of health to the effective price of health is equal for all family members and equal to the marginal utility of wealth.

2.3. The parents–child family

Adding a child to the husband–wife family model gives the following instantaneous strictly concave family utility function (time subscripts omitted):

\[ u = u(H_{m},H_{l},H_{c},Z), \]  

(20)

where \( H_{c} \) is child health, developing over time according to the equation of motion,

\[ \frac{\partial H_{c}}{\partial t} = I_{c} - \delta_{c} H_{c}, \]  

(21)

and produced by the child’s parents by use of market goods \( M_{c} \) and parental time \( (h_{H_{c,m}} \text{ and } h_{H_{c,f}} \text{, respectively}) \) according to the production function:

\[ I_{c} = I_{c}(M_{c}, h_{H_{c,m}}, h_{H_{c,f}}; E_{H_{m}}, E_{H_{f}}). \]  

(22)

The time restriction for each parent (husband and wife, respectively) then becomes

\[ \Omega_{i} = h_{w,i} + h_{Z,i} + h_{H_{m},i} + h_{H_{l},i} + h_{H_{c},i} + h_{S,i} + h_{S_{c,i}} \quad i = m,f \]  

(23)

where \( h_{S_{c,i}} \) is time taking care of a sick child for parent \( i \), and where \( \partial h_{S_{c,i}}/\partial H_{c} < 0 \) and \( \partial^{2} h_{S_{c,i}}/\partial H_{c}^{2} > 0. \)\(^{15} \) The family problem will now be extended to choose the paths of \( M_{m}, M_{f}, M_{c}, \) and \( Z \) in order to

Max \( U = \int_{t}^{T} e^{-\theta t} u(H_{m},H_{l},H_{c},Z) \)

such that

\[ \frac{\partial H_{j}}{\partial t} = I_{j} - \delta_{j} H_{j} \quad \text{for } j = m,f,c \]

\[ \frac{\partial W}{\partial t} = rW + \omega_{m}(H_{m},E_{w,m}) h_{w,m} + \omega_{l}(H_{l},E_{w,f}) h_{w,f} + B - p(M_{m} + M_{f} + M_{c}) - qX \]

\[ \Omega_{i} = h_{w,i} + h_{Z,i} + h_{H_{m},i} + h_{H_{l},i} + h_{H_{c},i} + h_{S,i} + h_{S_{c,i}} \quad \text{for } i = m,f \]

\( H_{l}(0) \) given for \( j = m,f,c \)

\(^{15} \)Thus, it is assumed that the parents themselves are taking care of their sick child. In reality, however, they may have other options.
\[ H_i(T) \leq H_{\min} \quad \text{for at least one of } j = m, f, c \]
\[ W(T) \geq 0, \quad W(T)^\# \lambda_W(T) = 0 \]
\[ T \text{ free} \]
\[ X_M \geq 0 \quad \text{for all } t \in [0, T], \quad j = m, f, c. \tag{24} \]

As in the previous case, \( T \) is the ‘lifetime’ of the parents–child family.\(^{16}\)

Solving the maximisation problem in Eq. (24) adds the marginal condition (see Appendix A, condition A18),
\[
\frac{\partial u}{\partial H_i} = \frac{\partial u}{\partial H_c} = \frac{\sigma_i(\delta_i + r - (\partial u_i^H/\partial H_i)W_i)}{\sigma_i}\left[\psi_i/\lambda_W (\partial h_{W,c}/\partial H_c) - (\psi_i/\lambda_W (\partial h_{W,c}/\partial H_c))\right] i = m, f, c
\]
\[
\tag{25}
\]
to the one in Eq. (18). The net effective marginal cost of adult health capital is the same as in condition (18). Net effective marginal cost of child health is equal to the user cost of child health capital less the marginal investment benefit of child health, which is the summation of the monetary value of the change in time taking care of a sick child for father and mother, respectively, for a marginal change in child health. Condition (19) is now extended to
\[
\lambda_W = \lambda_{H_m}/\pi_m = \lambda_{H_i}/\pi_i = \lambda_{H_c}/\pi_c, \tag{26}
\]
\[
\text{implying that the family invests in health until the rate of marginal utilities of (lifetime) health to effective price of health for all family members is equal and equal to the marginal utility of wealth. The family will not try to equalise the amount of health capital between family members.}
\]

Rearranging condition (26) gives that \( \lambda_{H_c} = \lambda_W \pi_c \), implying that poor families (where the wealth restriction is binding) value a marginal change in child health higher than rich families, and that families for who the wealth constraint is not binding (\( \lambda_W = 0 \)) has a zero marginal utility of child health. Further, it implies that a child with unhealthy parents can be expected to have lower health compared with a child with healthy parents, because resources have to be spent on increasing the health of the unhealthy parents to achieve condition (25).

\(^{16}\) Even though not directly included in the models above, for simplicity, an individual’s lifetime may be seen as consisting of three time periods. In the first time period, the individual is a child in a parents–child family; in the second period, he or she is a parent in the parents–child family; and in the third period, he or she is a husband or wife in the husband–wife family. His (her) initial health in the first period is his (her) inherited amount of health capital. Initial health in the second period is his (her) terminal amount of health in the first period and so on. This imply that his (her) health as old (in the third period) is determined by inherited health, investments made by his (her) parents during his (her) childhood (first period), and by own and spouse’s actions during adulthood (second and third period), given the exogenous variables.
3. Graphic illustrations of the model

3.1. The optimal distribution of family health

The maximisation problem in Eq. (24) is illustrated in Fig. 2, assuming a family consisting of one parent and one child, and for a given amount of \( Z \). Parent health \((H_i)\) is measured on the vertical axis and child health \((H_c)\) on the horizontal axis. The slope of the production possibility curve \((AB)\) is given by the right hand side of Eq. (25), i.e., the ratio of marginal net effective cost of adult health and marginal net effective cost of child health. \(UU\) is an indifference curve representing the level of family utility, with the slope given by the left hand side of Eq. (25).

The production possibility curve \(AB\) gives all possible combinations of child and parent health, given family resources in terms of time and initial wealth, and given the user cost of capital (net of monetary benefit). The shape of the production possibility curve is partly determined by the fact that as health is increased, more healthy time is allocated to market work, which increases family income.

![Fig. 2. An illustration of the solution to the maximisation problem in Section 2 given by the marginal condition in Eq. (25). This illustration assumes a family consisting of one parent and one child, and is drawn for a given level of \( Z \). Parent health \((H_i)\) is measured on the vertical axis and child health \((H_c)\) on the horizontal axis.](image)
If all time and wealth were allocated to the production of child health, the distribution of health would be given by point \( B \). Producing one amount of parent health using some of total family time and wealth would reduce adult sick time and increase family income, allowing child health to increase as well. It would be possible to increase both child and adult health at the same time (given time, wealth and user cost of capital) until health states defined by point \( D \) were reached. Then, increased adult health would be so ‘expensive’ that child health had to be reduced to make additional investments in adult health possible. If child health was reduced enough to allow for the production of adult health given by point \( F \), the parent would have to spend so much time taking care of the sick child that income would no longer be enough to make gross investments compensating for the depreciation, and child and adult health would both fall.

For completely selfish parents, child health per se would not enter the family utility function, so indifference curves would be horizontal. Maximising utility (for given \( Z \)) subject to the production possibility set (budget constraint) would then give point \( F \) in Fig. 2. This shows that even a selfish parent would invest in child health, but only because child health affects family income (the investment aspect of child health). However, because of altruism, adult utility will increase as child health increases. For a completely altruistic parent, indifference curves would be vertical. Maximising family utility then implies that the parent would be willing to invest in child health until point \( D \) is reached. Thus, altruism would be effective along the production possibility curve from \( F \) to \( D \). The location of point \( P \) on the production possibility curve between \( F \) and \( D \) will depend on the parent’s degree of altruism toward the child; ceteris paribus, a more altruistic parent would choose a point closer to \( D \) than a less altruistic parent.

If point \( E \) represents the endowed amounts of health capital at the beginning of period \( t \), the adult would invest in both own and child health until point \( P \) was reached. However, assume that adult health cannot be increased because of, for example, lack of treatment. Then the adult would maximise utility by investing in child health until point \( O \) is reached.

If, at the beginning of time \( t \), their endowed amounts of health capital were as represented by point \( O \), then the parent would invest in own health only and let child health depreciate until point \( P \) was reached. It may then look as if the parent underinvested in child health, but this would be a utility maximising behaviour given the restrictions he or she were faced with.

Because the health of one of the individuals could be increased without decreasing the health of the other, points on the segments \( AF \) and \( BD \) (in Fig. 2) do not represent stable situations. Health given by points on these positive sloping parts of the production possibility curve would only be stable if additional gross investments in health, to offset depreciation, could not be made because of, for example, lack of treatment. In this paper, it is assumed that the individuals always have this possibility, i.e., the analyses focus on the negatively sloping part of the production possibility curve (for a constant level of \( Z \)).
3.2. Effects of changes in exogenous variables on optimal family health

An increased rate of depreciation, $\delta$ (or an increased coinsurance rate in the health insurance system, increased $p$ that increases $\pi$) would affect family health by increasing the net cost of health capital. If both child and parent depreciation rates increased, but $\delta_c$ increased more than $\delta_m$, the point indicating optimal health would move from $P$ to $P'$ in Fig. 3. The income effect would decrease both child and parent health, but the substitution effect would increase parent health while decreasing child health. The total effect would be a reduction in child health, while the effect on parent health would be ambiguous. Using Fig. 3 to analyse the effect of a reduction in the coinsurance rate for medical care utilisation by children (aimed to increase child health), the effect on child health is positive but the effect on parent health is ambiguous. Because the cost of child health is reduced, resources previously used to produce child health can now partly be spent on adult health production (and partly on the production of other consumption commodities). Health related information will have a similar effect. Increased health related

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{This figure illustrates the effects of changes in the costs of health capital. Parent health ($H_p$) is given by the vertical axis and child health ($H_c$) by the horizontal axis. The initial production possibility curve is given by $AB$; $UU$ and $UU'$ are family indifference curves; and initial optimal health is given by point $P$. An increased cost of health, where the cost of child health increases more than the cost of parent health, will give the new production possibility curve $AB'$, and a new equilibrium given by point $P'$. The total effect on family health can be divided into an income effect (the move from $P$ to $P'$) and a substitution effect (the move from $P'$ to $P''$).}
\end{figure}
information will make the parent more productive in producing new health. This increase in $E_{Hi}$ will decrease both $\pi_1$ and $\pi_2$.

If the value of parent time is set equal to his or her wage rate (i.e., $\varphi_i / \lambda_{wi} = \omega_i$, $i = m, f$) and if some insurance exists, covering $x\%$ of losses due to taking care of a sick child, the net cost of child health will be:

$$\pi_c \left( \delta_c + r - \left( \frac{\partial \pi_c}{\partial t} \right) / \pi_c \right) - \left[ -\omega_a \left( \frac{\partial h_{Sc,sc}}{\partial H_c} \right) (1 - x) - \omega_f \left( \frac{\partial h_{Sc,sc}}{\partial H_c} \right) (1 - x) \right].$$

For $x > 0$, the net cost of child health in Eq. (27) is higher than the net cost of child health in Eq. (25) (the denominator on the right hand side), implying a lower optimal level of child health. Increasing $x$ will decrease the monetary value of investments in child health (increasing the net cost of child health) because an increased rate of compensation will reduce the incentive to invest in child health. The effect on family health by an increase in $x$ is shown in Fig. 4. Optimal health

![Fig. 4](image)

Fig. 4. This figure illustrates the effects of an increase in the rate of compensation for income losses due to taking care of a sick child ($x$) and an increase in transfer payment ($B$), respectively. The increase in the level of compensation will increase the slope of the production possibility curve from $AB$ to $AB'$, increasing the net cost of child health ($H_c$). Optimal health will move from point $P$ to $P'$. The effect on child health will be negative, while the effect on parent health ($H_i$) is ambiguous. An increase in transfer payment will increase family resources available for family production of health and other commodities, leaving the ratio of net cost of health capital unchanged. Family initial production possibilities are given by $AB$, and optimal health by point $P$. An increase in transfer payment will shift the production possibility curve to the right, and optimal health to $P''$. 
will move from $P$ to $P'$. The effect on child health is negative while the effect on parent health is ambiguous. The reason for this reduction in child health is that the parent cannot increase family income as much as before by investing in child health after $x$ is increased. An increase in the compensation for losses due to own (parent) illness will have a similar effect, but the reduction in $H_i$ due to the increase in the level of compensation will now partly be offset by the reduction in the wage rate caused by the parent’s reduced health.

An increase in transfer payment ($B$) has no effect on the marginal condition in Eq. (25), but it will have an effect on family health. As shown in Fig. 4, an increase in $B$ will shift the production possibility curve to the right. As shown, an increase in $B$ will increase both parent and child health, i.e., the point of optimal health will move from $P$ to $P''$. As Fig. 4 is drawn, the increase in $B$ is assumed to have no effect on the slope of the production possibility curve. However, it may be the case that an increase in $B$ affects the parent’s time allocation decision, for example decreasing the incentive for market work. Such a decrease in $h_{w,i}$ will then lead to a flatter production possibility curve.

4. Discussion and concluding remarks

This paper has extended the individualistic demand for health model, the Grossman model, by deriving a model of the family as producer of health. This is an important extension because it makes it possible to analyse the influence that other family members may have on an individual’s health related behaviour and to analyse differences in health and health care utilisation between children.

This section summarises the conclusions and discusses the implications of some of the assumptions and simplifications made in the paper. It ends with a discussion of some policy implications of the model.

4.1. Conclusion

The purpose of this paper was to develop a demand for health model that takes into account characteristics and behaviour of other family members’ on an individual’s health and health care utilisation. This was done by treating each family member as a producer of his own and other family members’ health.

The extended model, the family-as-producer-of-health model, has several implications. The main finding is that the family will not try to equalise marginal benefits and marginal costs of health capital for each family member. Instead, they will invest in health until the rate of marginal consumption benefits equals the rate of marginal net effective costs of health capital, or, in other words, family members will invest in health until the rate of marginal (lifetime) utility of health
to the effective price of health is equal for all family members (and equal to the marginal utility of wealth). The net effective cost of health capital equals the user cost of capital less the marginal investment benefit of health capital. Variables such as the level of compensation in the social insurance system, the effective price of care, health related information, and transfer payments will all affect the family production possibility set, and, therefore, the optimal level and distribution of family health.

Results related to individual health show that an increased rate of depreciation will decrease the individual’s level of health; that when an individual’s wage rate becomes more sensitive to differences in health, the individual will invest more in health; and that the more restricting the time constraint is, the higher the individual’s valuation of time will be and the more the individual invest in health.

Regarding child health, it is shown that poor families (where the wealth restriction is binding) value child health higher than rich families and that families where the wealth constraint is not binding have a zero marginal utility of child health. Further, a child with unhealthy parents can be expected to have lower health compared with a child with healthy parents, because resources have to be spent on increasing the health of the unhealthy parents for the marginal condition to be fulfilled. It is also shown that even a selfish parent will invest in child health because child health affects family income (the investment aspect of child health).

By assuming that adult health cannot be increased because, for example, lack of treatment, it is shown that the adult will maximise utility by ‘overinvesting’ in child health. If, on the other hand, their endowed amounts of health capital were such that the child’s health was above and the parent’s health below what is regarded as optimal, the parent will invest only in own health and let child health depreciate until optimum is reached.

The effects of changes in some exogenous variables on optimal family health were also considered in the paper. An increased rate of depreciation, an increased coinsurance rate in the health insurance system, or an increase in health related information will affect family health by increasing the net cost of health capital.

4.2. Common preferences

The model derived in this paper is based on the assumption of common preferences (Becker, 1991). As shown by recent empirical work, it may not be a proper description to assume that spouses have common preferences and to assume that who has the control of resources has no impact on how these resources are allocated. Empirical work shows, for example, that the distribution of non-earned income does matter. Thomas (1990) found that unearned income in the hands of a mother had a bigger effect on her family’s health than income under the control of a father. Thomas (1994) also examined the relationship between
parental education and child height. He found that the education of the mother had a bigger effect on her daughters' height, while paternal education had a bigger impact on his sons' height. He also found that as the woman's power in the household allocation process increased\textsuperscript{17}, she became more able to assert her preferences and direct more resources towards commodities she cares about.

Haddad and Hoddinott (1994) used a non-cooperative bargaining model to examine the impact of the intrahousehold distribution of income on the anthropometric status (height-for-age) of boys relative to girls. They found that an increase in female share of household income seemed not to be gender neutral; boys would gain relative to girls.

Several models of family behaviour have been suggested in the literature (for surveys, see Lundberg and Pollak, 1996 and Bergstrom, 1997). McElroy and Horney (1981; see also Manser and Brown, 1980) suggested a model where the distribution of resources within the family is seen as the outcome of a cooperative Nash bargaining process. According to this bargaining model, the objective of the spouses is to maximise a utility–gain production function, defined as the product of the husband's gain and the wife's gain from marriage. These gains from marriage will decrease when utility as single (the threat point) increases.

Then, using a Nash bargaining model to describe family behaviour, the effect on family health of an increase in the non-earned income by the wife is illustrated in Fig. 5. Male health ($H_m$) is given by the horizontal axis, and female health ($H_f$) by the vertical axis. An increase in the non-earned income by the wife will shift the production possibility curve outwards as in the common preference case in Fig. 4. However, in this case, an increase in non-earned income will also affect the iso–gain product curve (the bargaining analogue of indifference curves) through its effect on the threat-point. If utility as single is the actual threat-point, her bargaining power will increase, because the increase in her non-earned income will increase her utility as single. The iso–gain product curve will twist from $NN'$ to $N''N''$. Compared to the common preference model, the increase in female health will be larger, $H'_f - H_f$ compared to $H'_f - H_f$. The increase in the mother's non-earned income may also increase child health either because she cares more about child health than the father does, or because she prefers more healthy goods than he does. In the last case, not only child health will increase, but the health of all family members. Thus, according to the Nash bargaining model, family decisions are the outcome of some bargaining process and family demands will depend not only on prices and total family income, but also on the determinants of the threat points\textsuperscript{18}.

\textsuperscript{17} As indicators of power in the household allocation decisions the study used nonlabour income, opportunities outside the home, and relative educational status.

\textsuperscript{18} A Nash bargaining family demand for health model is formally derived in Bolin et al. (1999b), which also contains some additional results.
Fig. 5. An illustration of how distribution is determined according to the Nash bargaining model. Male health \( (H_m) \) is given by the horizontal axis, and female health \( (H_f) \) on the vertical axis. An increase in the non-earned income by the woman will shift the production possibility curve out as in the common preference case in Fig. 4 (the increase in transfer payment). However, in this case, an increase in non-earned income will also affect the iso-gain product curve (the bargaining analogue of indifference curves) through its effect on the threat-point. The iso-gain product curve will twist from \(NN\) to \(NN'\). Compared to the common preference model, the increase in female health will be larger, \(H_f - H_f'\) compared to \(H_f - H_f\). If her threat-point is unaffected by this increase in non-earned income, the new iso-gain product curve is given by the dashed curve \(NN'\).

4.3. Other assumptions and simplifications

In order to focus on the implications of extending the individual demand-for-health model to a family model, the model was derived assuming complete certainty. An individual (or a family) may face three types of uncertainty in health. First, there is uncertainty as to the current size of the health capital (a characteristic which the individual shares with his or her professional health care agents). Second, there is uncertainty about the rate of depreciation of the health capital. Third, there is uncertainty about the effects of the various inputs in the health production function on the health capital. The first type of uncertainty does not seem to have been introduced in any formal models. The second and/or the third types have been included in various ways in formal models developed by Cropper (1977), Dardanoni and Wagstaff (1990), Selden (1993), Chang (1996), Liljas (1998), and Picone et al. (1998). In an uncertain world, risk-averse individuals make larger investments in health and have greater (expected) health stocks than
they would in a perfectly certain world. This result is quite in accordance with the discussion in Grossman (1972a,b), even though it was not formally proved, since the original Grossman model ruled out uncertainty for the sake of simplicity. A portfolio approach to health behaviour has been discussed by Dowie (1975) and, more formally, by Horgby (1997). Their basic message is that the individual should diversify his or her health investment activities, if there is uncertainty about the effects on health capital of various measures intended to improve health.

Some implications of these results for the model developed in this paper seem to be worth noticing. The (expected) total health capital of the family would be larger than in the certain case. With common preferences, the relative distribution of (expected) health capital among family members would remain the same as in the certain case. With non-common preferences, however, the relative distribution may change, since family members may then have different attitudes towards risk and uncertainty. For the same reason, the optimal portfolio of health investments may be quite different for different family members, not only because of the characteristics included in the certain case developed in this paper but also depending on diverging attitudes towards risk among family members. Thus, this phenomenon may contribute to explaining differing morbidity and mortality expectations and experiences among family members.

A further simplification made in the paper is that important family related decisions such as family formation (i.e., marriage or divorce), family size and inter sibling allocation of resources were not considered. Those issues are discussed by Becker (1991), Becker and Lewis (1973) and Becker and Tomes (1976), and give rise to issues such as assortative marriage (that equally healthy or wealthy individuals marry each other), the interaction between quantity and quality of children, and whether the transfers of resources from parents to children are based on efficiency or equity considerations.

4.4. Policy implications

The model presented in this paper has some important policy implications when discussing and analysing differences in health and health care utilisation among individuals and for a single individual over time. It was shown that variables such as other family members’ health, preferences, education, income, etc., are important for an individual’s stock of health and health care utilisation (i.e., health related behaviour). It was further shown that an individual’s present health and health care utilisation is the result of investments made earlier in life by him or herself, and by his or her parents.

Grossman suggested that the simplest way to introduce uncertainty might be to let a given consumer face a probability distribution of depreciation rates in every period (Grossman 1972a,b). However, none of the studies so far has employed such an approach.
Consequently, the individual’s family situation has to be considered when formulating prevention, treatment, rehabilitation and educational health programmes. But family variables also have to be included in the evaluation of such programmes and in analyses of reasons to differences in health and health care utilisation.

Naturally, the extended theoretical model also calls for new empirical estimations, which can increase our knowledge about health behaviour both per se and in policy-relevant contexts. Some preliminary results are available in Bolin et al. (1998a), who analyse the demand for health and health care in Sweden 1980/81 and 1988/89, using a Swedish panel data set and considering both the dynamic character of the Grossman model and the impact of the family structure.

Acknowledgements

I am grateful for the helpful comments and suggestions on earlier drafts from Björn Lindgren, Kristian Bolin, Inga Persson, participants at the seminar in political economy at the Department of Economics, Lund University, and two referees. This research was supported by grants from Vardalstiftelsen and from the Swedish National Institute of Public Health, which is gratefully acknowledged.

Appendix A

To solve the dynamic optimisation problem in Section 2, optimal control theory (Chiang, 1992) is used. The problem is formulated as an equality constraint problem where time is binding in each period, but where the family is free to borrow and lend during the family lifetime. It contains given initial points, but a variable terminal point; i.e., a horizontal-terminal line problem. The Lagrangian for solving this problem is (Chiang, 1992, p. 276):

\[ L = u(H_m, H_f, H_c, Z) e^{-\theta t} + \lambda_{H_m} [I_m(M_m, H_m,m, H_{H,m}; E_{H,m}, E_{H,f}) - \delta_m H_m] + \lambda_{H_f} [I_f(M_f, H_{H,f}; E_{H,f}) - \delta_f H_f] + \lambda_{H_c} [I_c(M_c, H_{H,c}; E_{H,c}) - \delta_c H_c] + \lambda_{w} [rW + \omega_m(H_m, E_{w,m}) h_{w,m} + \omega_f (H_f, E_{w,f}) h_{w,f} - p(M_m + M_f + M_c) - qX] + \varepsilon_m [\Omega - h_{w,m} - h_{Z,m} - h_{H,m}H_m - h_{H,m} - h_{H,c} - h_{S,m} - h_{S,c}] + \varepsilon_f [\Omega - h_{w,f} - h_{Z,f} - h_{H,f}H_f - h_{H,c} - h_{S,f} - h_{S,c}] \]  

(A1)
F.O.C. (interior solution)
\[
\frac{\partial l}{\partial M} = \frac{\partial l}{\partial X} = 0 \quad \text{for all } t \in [0, T]
\]
\[
\frac{\partial l}{\partial \varphi_m} = \frac{\partial l}{\partial \varphi_i} = 0
\]
\[
\frac{\partial H_m}{\partial t} = \frac{\partial l}{\partial H_m} \quad \text{equation of motion for the state variable } H_m
\]
\[
\frac{\partial H_i}{\partial t} = \frac{\partial l}{\partial H_i} \quad \text{equation of motion for the state variable } H_i
\]
\[
\frac{\partial W}{\partial t} = \frac{\partial l}{\partial W} \quad \text{equation of motion for the state variable } W
\]
\[
\frac{\partial \lambda_{Hm}}{\partial t} = -\frac{\partial l}{\partial H_m} \quad \text{equation of motion for } \lambda_{Hm}
\]
\[
\frac{\partial \lambda_{Ht}}{\partial t} = -\frac{\partial l}{\partial H_t} \quad \text{equation of motion for } \lambda_{Ht}
\]
\[
\frac{\partial \lambda_{Hc}}{\partial t} = -\frac{\partial l}{\partial H_c} \quad \text{equation of motion for } \lambda_{Hc}
\]
\[
\frac{\partial \lambda_w}{\partial t} = -\frac{\partial l}{\partial W} \quad \text{equation of motion for } \lambda_w
\]
These F.O.C.’s then give:
\[
\frac{\partial l}{\partial M_j} = \lambda_{Hj} \frac{\partial I_j}{\partial M_j} - \lambda_w p = 0 \quad j = m,f,c
\]
\[
\frac{\partial l}{\partial X} = e^{-\theta t} \frac{\partial u}{\partial Z} \frac{\partial Z}{\partial X} - \lambda_w q = 0
\]
\[
\frac{\partial l}{\partial \lambda_{Hj}} = I\left(M_j, h_{H_j,m}, h_{H_j,f}, E_{H,m}, E_{H,f}\right) - \delta_j H_j \quad j = m,f,c
\]
\[
\frac{\partial l}{\partial \lambda_w} = rW + \omega_m (H_m, E_{w,m}) h_{w,m} + \omega_f (H_f, E_{w,f}) h_{w,f} - p(M_m + M_f + M_c) - qX
\]
\[
\frac{\partial l}{\partial H_i} = \frac{\partial u}{\partial H_i} e^{-\theta t} - \lambda_{Ht} \delta_i + \lambda_w h_{w,i} \frac{\partial \omega_i}{\partial H_i} - \varphi \frac{\partial h_{sc,i}}{\partial H_i}
\]
\[
\frac{\partial l}{\partial H_c} = \frac{\partial u}{\partial H_c} e^{-\theta t} - \lambda_{Hc} \delta_c - \varphi \frac{\partial h_{sc,c}}{\partial H_c} - \varphi \frac{\partial h_{sc,f}}{\partial H_c}
\]
\[
\frac{\partial l}{\partial W} = \lambda_w r = -\frac{\partial \lambda_w}{\partial t}
\]
Rewriting Eq. (A3) gives:
\[
\lambda_{Hj} = \lambda_w p / (\partial I_j / \partial M_j) = \lambda_w \pi_j \quad j = m,f,c
\]
and the time derivative of Eq. (A10):
\[
\frac{\partial \lambda_{Hj}}{\partial t} = \frac{\partial \lambda_w}{\partial t} \pi_j + \lambda_w \frac{\partial \pi_j}{\partial t} \quad j = m,f,c
\]
Solution to the single person family problem: setting Eq. (A7) equal to Eq. (A11), for \( j = i \), gives
\[
\left(\frac{\partial u}{\partial H_i} e^{-\theta t} - \lambda_{Ht} \delta_i + \lambda_w h_{w,i} \frac{\partial \omega_i}{\partial H_i} - \varphi \frac{\partial h_{sc,i}}{\partial H_i}\right)
\]
\[
\left(\frac{\partial \lambda_w}{\partial t} \pi_i + \lambda_w \left(\frac{\partial \pi_i}{\partial t}\right)\right),
\]
\[ (A12) \]
and using Eqs. (A10) and (A9) to substitute for \( \lambda_{H,m} \) and \( -\partial \lambda_w / \partial t \) gives (subscript \( i \) omitted)

\[
(\partial u / \partial H) e^{-\theta t} - \lambda_w \pi \delta + \lambda_w \ h_w (\partial \omega / \partial H) - \varphi (\partial h_{s, f} / \partial H)
= \lambda_w \ r \pi - \lambda_w \ (\partial \pi / \partial t) \quad (A13)
\]

Re-arranging Eq. (A13) and adding the time subscript gives the marginal condition in Eq. (8), i.e.,

\[
(\exp(-\theta t / \lambda_{i,w}) \partial u / \partial H_i + \partial \omega_i / \partial H_i - \varphi_i (\partial h_{s, i,f} / \partial H_i)
= \pi_i \left[ \delta_i + r - (\partial \pi_i / \partial t) / \pi_i \right] . \quad (A14)
\]

Solution to the husband–wife family problem: use Eqs. (A9)–(11) to write Eq. (A7) as

\[
(\partial u / \partial H_i) e^{-\theta t} - \lambda_w \pi_i \delta_i + \lambda_w \ h_w (\partial \omega_i / \partial H_i) - \varphi_i (\partial h_{s, i,f} / \partial H_i)
= \lambda_w \ r \pi_i - \lambda_w \ (\partial \pi_i / \partial t) , \quad i = m,f. \quad (A15)
\]

Set \( i \) equal to \( m \) and \( f \), respectively, and divide the expression for \( m \) with the one for \( f \) to obtain the marginal condition in Eq. (18),

\[
\frac{\partial u / \partial H_m}{\partial u / \partial H_f} - \frac{\pi_m (\delta_m + r - (\partial \pi_m / \partial t) / \pi_m) - [(\partial \omega_m / \partial H_m) h_{s,m} - (\varphi_m / \lambda_w) (\partial h_{s,m} / \partial H_m)]}{\pi_f (\delta_f + r - (\partial \pi_f / \partial t) / \pi_f) - [(\partial \omega_f / \partial H_f) h_{s,f} - (\varphi_f / \lambda_w) (\partial h_{s,f} / \partial H_f)]} . \quad (A16)
\]

Solution to the parents–child family problem: use Eqs. (A9)–(11) to write Eq. (A8) as:

\[
(\partial u / \partial H_i) e^{-\theta t} - \lambda_w \pi_i \delta_i - \varphi_i (\partial h_{s,c,m} / \partial H_c) - \varphi_i (\partial h_{s,c,f} / \partial H_c)
= \lambda_w \ r \pi_i - \lambda_w \ (\partial \pi_i / \partial t) \quad (A17)
\]

The marginal condition in Eq. (25) is then obtained by dividing Eq. (A15) for \( i = m \) with Eq. (A17), i.e.,

\[
\frac{\partial u / \partial H_m}{\partial u / \partial H_c} - \frac{\pi_m (\delta_m + r - (\partial \pi_m / \partial t) / \pi_m) - [(\partial \omega_m / \partial H_m) h_{s,m} - (\varphi_m / \lambda_w) (\partial h_{s,m} / \partial H_m)]}{\pi_f (\delta_f + r - (\partial \pi_f / \partial t) / \pi_f) - [(\partial \omega_f / \partial H_f) h_{s,f} - (\varphi_f / \lambda_w) (\partial h_{s,f} / \partial H_f)]} . \quad (A18)
\]

References

Bolin, K., Jacobson, L., Lindgren, B., 1999. The family as the health producer — the case when family members are Nash-bargainers. Lund University, Departments of Economics and Community Medicine (mimeo).