Fixity of Hierarchy, Effort Level, and Human Capital in Promotion Probability

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Introduction

- Promotion appears to be the most important form of pay for performance in most organisations, especially in hierarchical, white-collar firms.
- Promotion is the primary mean by which workers can increase their long-run compensation (McCue 1992; Lazear 1992).
- Promotion generates substantial motivation in many settings.
- No strong pay for performance within jobs increases the apparent importance of promotion for organisational incentives (Hedström 1987).

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- Promotion, more often than not, occurs in the setting of a tournament.
- In general, people may implicitly compete for a limited number of indivisible slots.
- A key assumption in tournament model: The structure and number of jobs in a hierarchy are relatively fixed.
- Contribution of this paper:
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  - Extends the conventional tournament model to multi-person settings.
  - Analyses how incentives vary with promotion rates.
  - Empirically tests how fixity of hierarchy influence promotion probability.

Introduction

- Related literature.
- Model of multi-person tournaments.
- Data
- Fixity of hierarchy
- Econometric model
- Results
- Conclusions.

Related Literature

- A large number of previous research on promotion probability focuses on the role of schooling in promotion practice.
- Schooling is positively related to promotion probabilities - Baker, Gibbs, and Holmstrom (1994a,b), McCue (1996), and Lluis (2005).
- Schooling is positively related to the wage even after controlling for experience and job assignment - Medoff and Abraham (1980, 1981) and Baker, Gibbs, and Holmstrom (1994a,b)
- Positive relation between schooling and the return to experience - Farber and Gibbons (1996), Rubinstein and Weiss (2007), and Habermalz (2006).
- Much of wage growth during careers is the result of accumulation of human capital - Medoff and Abraham (1980, 1981).

Multi-person Tournaments

- To model promotions with probabilities different from $\frac{1}{2}$, more general competitions among n contestants for k equal prizes.
- The promotion probability is $p = k/n$.
- A symmetric Nash equilibrium is assumed.
- To win the contest a worker must place k^{th} or higher, which amounts to beating at least $n-k$ co-workers in pair-wise output comparisons, *i.e.*

$$\begin{aligned} p\left(q_i > q_g\right) &= p\left(\mu_i + \varepsilon_i > \mu_g + \varepsilon_g\right) \\ &= p\left(\varepsilon_g < \mu_i - \mu_g + \varepsilon_i\right) \\ &= F\left(\mu_i - \mu_g + \varepsilon_i\right). \end{aligned} \tag{1}$$

Multi-person Tournaments

- To finish exactly j^{th} from the top out of a field of n contestants, one must beat $n-j$ opponents and lose to $j-1$ opponents. The probability of doing so for any given partition of competitors into the $n-j$ the worker beats and the $j-1$ the worker loses to, conditional on $i's$ luck ε_i, is $F\left(\mu_i-\mu^*+\varepsilon_i\right)^{n-j}\left(1-F\left(\mu_i-\mu^*+\varepsilon_i\right)\right)^{j-1}$.
- The number of ways to choose $n-j$ elements from a collection of $n-1$ is

$$\binom{n-1}{n-j}=(n-1)!/\left(n-j\right)!\left(j-1\right)!. \tag{2}$$

- The probability of placing exactly j^{th} from the top, conditional on ε_i, is:

$$\binom{n-1}{n-j}F\left(\mu_i-\mu^*+\varepsilon_i\right)^{n-j}\left(1-F\left(\mu_i-\mu^*+\varepsilon_i\right)\right)^{j-1}. \tag{3}$$

Multi-person Tournaments

- The probability of promotion conditional on ε_i equals the sum of the conditional probabilities of placing exactly 1^{st} through k^{th}. Integrating out ε_i then gives the unconditional probability of promotion:

$$p\left(\mu, \underline{\mu}^*\right) = \sum_{j=1}^{k} \binom{n-1}{n-j} \int_{-\infty}^{\infty} F\left(\mu - \mu^* + \varepsilon\right)^{n-j} \left(1 - F\left(\mu - \mu^* + \varepsilon\right)\right)^{j-1} f\left(\varepsilon\right) d\varepsilon. \quad (4)$$

- $p\left(\mu, \underline{\mu}^*\right)$ is constant if $p = 0$ or 1 $(k = 0$ or $n)$, and the marginal probability of effort (MPE) and incentives are zero. To solve for the MPE, differentiate and substitute in the symmetric Nash equilibrium condition $\mu_i = \mu^*$:

$$\frac{\partial p}{\partial e} = \sum_{j=1}^{k} \binom{n-1}{n-j} \int \left(\left(n-j\right) F\left(\varepsilon\right)^{n-j-1} \left(1 - F\left(\varepsilon\right)\right)^{j-1} - \left(j-1\right) F\left(\varepsilon\right)^{n-j} \left(1 - F\left(\varepsilon\right)\right)^{j-2} \right) f^2\left(\varepsilon\right) d\varepsilon. \quad (5)$$

Multi-person Tournaments

- The second expression is zero if $j = 1$, this equals:

$$\frac{\partial p}{\partial e} = \sum_{j=1}^{k} (n-j) \binom{n-1}{n-j} \int F(\varepsilon)^{n-j-1} (1 - F(\varepsilon))^{j-1} f(\varepsilon)^2 \, d\varepsilon$$
$$- \sum_{m=2}^{k} (n-(m-1)) \binom{n-1}{n-(m-1)} \tag{6}$$
$$\int F(\varepsilon)^{n-m} (1 - F(\varepsilon))^{m-2} f(\varepsilon)^2 \, d\varepsilon.$$

- Letting $j = m - 1$, all terms in the second sum cancel terms in the first, leaving the term $j = k$:

$$\frac{\partial p}{\partial e} = (n-k) \binom{n-1}{n-k} \int F(\varepsilon)^{n-k-1} (1 - F(\varepsilon))^{k-1} f(\varepsilon)^2 \, d\varepsilon. \tag{7}$$

- A contestant is actually trying to beat the $n - k - 1^{st}$ order statistic; *i.e.*, the random variable that is the $n - k - 1^{st}$ score (from the bottom) among competitors.

Multi-person Tournaments

- Order statistics/Performance to beat, κ. Subtracting off optimal effort of competitors gives the luck λ required to win, $\lambda = \kappa - \mu^*$, a random variable.
- With a symmetric Nash equilibrium effort terms cancel out, and λ is also an order statistic: the $n-k-1^{st}$ luck (from the bottom) among the $n-1$ competitors. If this random variable has *c.d.f.* $G(\lambda)$ and *p.d.f.* $g(\lambda)$, then for given luck ε, the probability of promotion equals:

$$\Pr(promoted \mid \varepsilon) = p(\mu + \varepsilon > \mu^* + \lambda \mid \varepsilon) = G(\mu - \mu^* + \varepsilon). \quad (8)$$

- Integrating over the luck distribution gives the unconditional probability of promotion,

$$p = \int G(\mu - \mu^* + \varepsilon) f(\varepsilon) d\varepsilon. \quad (9)$$

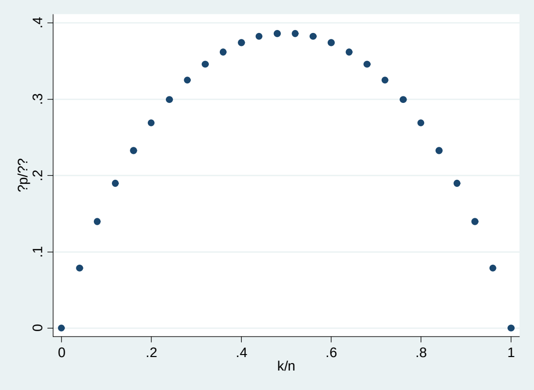
- Differentiating and substituting in $\mu = \mu^*$ gives

$$\frac{\partial p}{\partial \mu} = \int g(\varepsilon) f(\varepsilon) d\varepsilon. \quad (10)$$

Multi-person Tournaments

k/n	$\partial p/\partial e$
0/25	0
1/25	7.8613×10^{-2}
2/25	0.139 58
5/25	0.26897
10/25	0.37423
12/25	0.38621
13/25	0.38621
15/25	0.37423
20/25	0.26897
23/25	0.139 58
24/25	7.8613×10^{-2}
25/25	0

Multi-person Tournaments



Multi-person Tournaments

- When f is symmetric unimodal, the *MPE*, $\partial p/\partial\mu$, is equal in the two tournaments where k out of n are promoted and where $n-k$ out of n are promoted. These are any two tournaments with the same number of contestants and promotion probabilities of p and $1-p$.
- f is symmetric about $\frac{1}{2}$, and the density of $F(\varepsilon)$ in the first tournament is a reflection about $\frac{1}{2}$ of the density in the second, and the integrals that equal $\partial p/\partial\mu$ in the two cases must be equal.
- Intuitively, this is a contest among even competitors, and ε is distributed symmetrically.
- The probability of placing j^{th} from the top equals the probability of placing j^{th} from the bottom, and otherwise identical contests with probabilities symmetric about $\frac{1}{2}$ give the same incentives.

Multi-person Tournaments

- When f is symmetric unimodal, adding a contestant $(n$ to $n+1)$, while holding the number of winners fixed (k), decreases the *MPE*, $\partial p/\partial\mu$, for $k<\frac{1}{2}n$ (less than $\frac{1}{2}$ promoted), does not change $\partial p/\partial\mu$ for $k=\frac{1}{2}n$, and increases $\partial p/\partial\mu$ for $k>\frac{1}{2}n$ (more than $\frac{1}{2}$ promoted).
- For $p<\frac{1}{2}$, increasing n but not k increases the skew of the density of $F(\varepsilon)$, shifting mass away from $\frac{1}{2}$. Since f decreases away from $\frac{1}{2}$ the result follows.
- For $p>\frac{1}{2}$, increasing n but not k reduces the skew of the density of $F(\varepsilon)$, with the opposite effect.
- As the promotion becomes less or more likely (p approaches 0 or 1), more extreme good or bad luck is required to win or lose. Extreme luck is less likely than luck closer to zero, so that marginal effort is less likely to make a difference.

Multi-person Tournaments

- When f is symmetric unimodal, adding one more winner while keeping the number of contestants fixed increases the *MPE*, $\partial p/\partial \mu$, for $k < \frac{1}{2}(n-1)$, does not change $\partial p/\partial \mu$ for $k = \frac{1}{2}(n-1)$, and decreases $\partial p/\partial \mu$ for $k > \frac{1}{2}(n-1)$.
- Incentives rise as p rises from 0 to $\frac{1}{2}$, peak at $p = \frac{1}{2}$, and fall as p rises from $\frac{1}{2}$ to 0.
- This effect on incentives is symmetric with respect to the absolute difference of the promotion rate and $\frac{1}{2}$, $\left|p - \frac{1}{2}\right|$.
- Adding competitors or decreasing promotion slots can raise incentives if it brings p closer to $\frac{1}{2}$.

Data

- A major British financial sector firm with around 40,000 full-time employees and 20,000 part-time employees.
- Personnel and payroll archives referring to the firm's British operations from January 1989 to November 2001 - 155 monthly observations.
- Individual characteristics e.g. age, gender, marital status, salary, hierarchical grade, date of entry, performance rating etc.
- Both clerical and managerial grades. Explicit hierarchical structure. 14 levels.
- 2 grades excluded. 3-4 junior staff, 5-6 senior staff, 7-8 junior manager, 9-10 senior manager, 11-13 executive manager.
- Five different appraisal ratings: "outstanding" (5), "very good" (4), "satisfactory" (3), "not fully effective" (2), "unsatisfactory" (1).
- No rating: lag between a hire or promotion and the first appraisal in the new job.

Fixity of Hierarchy

- Testing fixity of hierarchy in aggregate level. Pooled individual data in grades on a yearly basis.
- Internal labour market:
    - No. of promotion into
- Promotion into rate, exit (promotion out of + quit) rate.

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  - No. of quit
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Fixity of Hierarchy

- Regression of P. on External Hiring and Exit Rates by Title

Promotion Rate	Coef.	Std. Err.	t	P>|t|
E. Hiring Rate	-.5070253	.0898473	-5.64	0.000
Exit Rate	.6707882	.1009921	6.64	0.000

- Regression of Adj. P. on External Hiring and Exit Rates by Title

Adj. P. Rate	Coef.	Std. Err.	t	P>|t|
E. Hiring Rate	-.5927631	.092532	-6.41	0.000
Exit Rate	.7152461	.1040098	6.88	0.000

Econometric Framework

- Testing the impact of changes in hierarchy on promotion probability.
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  - Promotion probabilities are potentially affected by external hiring rate and exit rate in the adjacent upper level.
  - Is performance rating correlated with external hiring rate and exit rate in the adjacent upper level?

Econometric Model

- The definition of a promotion:

$$Y_{it} = 1\left(L_{it} - L_{it-1} > 0\right). \tag{11}$$

- The promotion probability:

$$\begin{aligned} \Pr\left(Y_{it} = 1\right) &= \Phi(\beta_X \cdot X_{it-1} + \beta_J \cdot J_{t-1} + \beta_{L5} \cdot L_{5it-1} \\ &\quad + ...\beta_{L10} \cdot L_{10it-1} + \beta_H \cdot h_{t-1} + \beta_R \cdot r_{t-1} \\ &\quad + \beta_{HR} \cdot h_{t-1} \cdot r_{t-1} + \alpha_i^p) \end{aligned} \tag{12}$$

  - *X: individual specific attributes (gender, age, education, tenure within the firm etc.).*

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  - *α_i: unobserved heterogeneity.*

Results

Promotability	(1)	(2)
Firm Size	.87321	.88375
	(.1579536)	(.2087456)
Firm Size Change	.8786549	.7947893
	(.1580214)	(.1986754)
Age	−.0227905	−.0284227
	(.0005423)	(.0007853)
Tenure	.007358	
	(.0005122)	
Banking Certificate	0.276467	0.356428
	(.0132664)	(.0174683)

Results

Promotability	(1)	(2)
A Level	.0568377	.0667327
	(.0063802)	(.0043967)
University	.2446592	.3246483
	(.0193615)	(.0147638)
Postgrad	.2112779	.2382732
	(.0237057)	(.0238942)

Results

Promotability	(1)	(2)
Level 6	.2432197	.2232583
	(.0127404)	(.0116438)
Level 7	.1503348	.1307862
	(.012061)	(.015724)
Level 8	.3813616	.4126734
	(.0165188)	(.0202163)
Level 9	.3730026	.3906773
	(.0265089)	(.0290594)
Level 10	.444527	.478326
	(.0481467)	(.0503842)
Level 7 $\times$ Performance Ratings	.365134	.394268
	(.2356742)	(.2752513)

Results

Promotability	(1)	(2)
Gender	$-.06362$	$-.07139$
	(.0124737)	(.0146514)
Gender × Level 6	$-.046828$	$-.058423$
	(.017604)	(.015383)
Gender × Level 7	.0134634	.0147812
	(.0179543)	(.0197421)
Gender × Level 8	.0160687	.0187164
	(.0207322)	(.0250124)
Gender × Level 9	.014051	.018473
	(.030199)	(.0345679)
Gender × Level 10	.0173876	.0193218
	(.052388)	(.057685)

Results

Promotability	(1)	(2)
Performance Ratings	.6072137	.6789652
	(.0429923)	(.0484568)
N. L. Hiring Rate	−.6606578	−.6974456
	(.7510452)	(.7287542)
N. L. Exit Rate	3.352206	3.745643
	(.4626029)	(.4897854)
N. L. Exit Rate Square	−8.527467	−9.037291
	(.7839544)	(.86273462)
N. L. Hiring Rate × P. R.	.5647656	.5883472
	(.3898176)	(.4064563)
N.L. Exit Rate × P. R.	−1.997103	−2.254326
	(.2502864)	(.2786132)

Conclusions

- Marginal probability of effort only depends on k and n.
- Among standard individual specific human capital endowment variables, education and professional qualification are relatively more important.
- Fixity of hierarchy influences promotion outcomes.
- Effort level varies closely with fixity of hierarchy.

Future Work

- Initial condition problem.
- Endogenous sampling and attrition.
- The marginal effect.